Instructions: Complete each of the following exercises for practice.

1. Compute the directional derivative $D_{\mathbf{u}}f$ at point P in the direction of angle θ.
   (a) $f(x,y) = xy^3 - x^2; \quad P = (1,2), \quad \theta = \frac{\pi}{3}$
   (b) $f(x,y) = y\cos(xy); \quad P = (0,1), \quad \theta = \frac{\pi}{4}$
   (c) $f(x,y) = \sqrt{2x+3y}; \quad P = (3,1), \quad \theta = -\frac{\pi}{6}$

2. Find the directional derivative of f at the point P in the direction of $\mathbf{v}$.
   (a) $f(x,y) = \dfrac{x}{x^2+y^2}; \quad P = (1,2), \quad \mathbf{v} = \langle 3,5\rangle$
   (b) $f(u,v) = u^2e^{-v}; \quad P = (3,0), \quad \mathbf{v} = \langle 3,4\rangle$
   (c) $f(x,y,z) = x^2y + y^2z; \quad P = (1,2,3), \quad \mathbf{v} = \langle 2,-1,2\rangle$
   (d) $f(r,s,t) = \ln(3r+6s+9t); \quad P = (1,1,1), \quad \mathbf{v} = \langle 4,12,6\rangle$

3. Use the limit definition of the partial derivative to compute f_x and f_y for the functions $f(x,y)$ below.
   (a) $f(x,y) = xy^2 - x^2y$
   (b) $f(x,y) = \dfrac{x}{x+y^2}$

4. Compute all first order partial derivatives of the given functions.
   (a) $f(x,y) = x^4 + 5xy^3$
   (b) $f(x,y) = x^2y - 3y^4$
   (c) $f(x,t) = t^2e^{-x}$
   (d) $f(x,t) = \sqrt{3x+4t}$
   (e) $f(x,t) = \ln(x+t^2)$
   (f) $f(x,y) = x\sin(xy)$
   (g) $g(u,v) = (u^2v - v^3)^5$
   (h) $u(r,\theta) = \sin(r\cos(\theta))$
   (i) $f(x,y) = \dfrac{x}{(x+y)^2}$
   (j) $f(x,y) = \dfrac{x}{y}$
   (k) $L(x,y) = \dfrac{ax+by}{cx+dy}$
   (l) $F(x,y) = \displaystyle\int_{t=x}^{y} \cos(e^t)\ dt$
   (m) $u(x_1,x_2,\cdots,x_n) = \sin(x_1 + 2x_2 + \cdots + nx_n)$
   (n) $p(t,u,v) = \sqrt{t^4 + u^2\cos(v)}$
   (o) $f(x,y) = x^y$
   (p) $f(x,y,z) = x^3yz^2 + 2yz$
   (q) $f(x,y,z) = xy^2e^{-xz}$
   (r) $w(x,y,z) = \ln(x+2y+3z)$
   (s) $w(x,y,z) = y\tan(x+2z)$
   (t) $u(x,y,z) = x^{\frac{y}{z}}$
   (u) $R(p,q) = \arctan(pq^2)$
   (v) $w(u,v) = \dfrac{e^v}{u+v^2}$
   (w) $h(x,y,z,t) = x^2y\cos\left(\dfrac{z}{t}\right)$
   (x) $\varphi(x,y,z,t) = \dfrac{\alpha x + \beta y^2}{\gamma z + \delta t^2}$
   (y) $F(\alpha,\beta) = \displaystyle\int_{t=\alpha}^{\beta} \sqrt{t^3+1}\ dt$
   (z) $u(x_1,x_2,\cdots,x_n) = \sqrt{x_1^2 + x_2^2 + \cdots + x_n^2}$

5. Use implicit differentiation to compute $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$ for the following implicit surfaces.
   (a) $x^2 + y^2 + z^2 = 1$
   (b) $x^2 - y^2 + z^2 - 2z = 4$
   (c) $e^z = xyz$
   (d) $yz + x\ln(y) = z^2$

6. Compute all second order partial derivatives.
   (a) $f(x,y) = x^4y - 2x^3y^2$
   (b) $f(x,y) = \ln(ax+by)$
   (c) $g(r,\theta) = e^{-2r}\cos(\theta)$
   (d) $v(s,t) = \sin(s^2 - t^2)$

7. Compute the indicated partial derivatives.

(a) $f(x, y) = x^4y^2 - x^3y$; $\quad f_{xxx}$, $\quad f_{xyx}$

(b) $g(r, s, t) = e^r \sin(st)$; $\quad g_{rst}$

(c) $w(x, y, z) = \dfrac{x}{y + 2z}$; $\quad \dfrac{\partial^3 w}{\partial z \partial y \partial x}$, $\quad \dfrac{\partial^3 w}{\partial x^2 \partial y}$